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INTRODUCTION

The Institute for Computer Applications in Science and Engineering (ICASE) is operated at the Langley Research Center (LaRC) of NASA by the Universities Space Research Association (USRA) under a contract with the Center. USRA is a nonprofit consortium of major U. S. colleges and universities.

The Institute conducts unclassified basic research in applied mathematics, numerical analysis, and computer science in order to extend and improve problem-solving capabilities in science and engineering, particularly in aeronautics and space.

ICASE has a small permanent staff. Research is conducted primarily by visiting scientists from universities and from industry, who have resident appointments for limited periods of time, and by consultants. Members of NASA's research staff also may be residents at ICASE for limited periods.

The major categories of the current ICASE research program are:

a. Numerical methods, with particular emphasis on the development and analysis of basic numerical algorithms;

b. Control and parameter identification problems, with emphasis on effective numerical methods;

c. Computational problems in engineering and the physical sciences, particularly fluid dynamics, acoustics, and structural analysis;

d. Computer systems and software, especially vector and parallel computers.

ICASE reports are considered to be primarily preprints of manuscripts that have been submitted to appropriate research journals or that are to appear in conference proceedings. A list of these reports for the period April 1, 1984, through September 30, 1984, follows a brief description of research in progress in the next section.

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RESEARCH IN PROGRESS

Loyce M. Adams

Work concerning the convergence questions of parallel (multi-color) SOR was done in conjunction with H. Jordan (University of Colorado). We were able to construct multi-color SOR schemes (for a wide range of stencils used to discretize partial differential equations) with the same rate of convergence as the corresponding natural rowwise ordered SOR scheme. Currently, unanswered questions that were stimulated by this research, such as the convergence rate of multi-color SSOR, are being pursued.

Work is continuing on grouping orderings for two-dimensional stencils into equivalence classes with all orderings in the same equivalence class yielding the same asymptotic rate of convergence when used with the SOR method to solve systems of linear equations. We hope that this classification will aid in choosing an appropriate ordering for SOR to be used with a particular parallel computer architecture. Also, we anticipate that the techniques used to find these orderings can also be applied to the problem of finding optimal orderings for other parallel algorithms on parallel architectures. Discussions along these lines are continuing with J. Ortega (University of Virginia).

Work is beginning on the extension of these ideas to three-dimensional problems.

H. Thomas Banks and James M. Crowley

We have continued our efforts on techniques for the estimation of spatially varying parameters (stiffness, damping) in models for elastic systems. Our recent studies involve simultaneous determination of damping and stiffness in a cantilevered Euler-Bernoulli beam. We have successfully carried out numerical experiments based on theoretically sound (i.e., we can also prove convergence of our schemes) spline approximation schemes. In some

(but not all) cases, convergence of our iterative algorithms can be accelerated slightly with the use of regularization ideas.

H. Thomas Banks, Katherine A. Murphy, and Kazufumi Ito

We have investigated and compared several formulations of a problem for the estimation of spatially dependent (including discontinuous) coefficients and boundary parameters (material parameters in absorbing boundary conditions at one boundary, source and material parameters in elastic boundary conditions) in problems for hyperbolic systems that are typical of layered media inverse problems. The algorithms are based on either spline or tau-Legendre approximation schemes. Reports detailing some of our findings are currently in preparation.

H. Thomas Banks and I. Gary Rosen

We are continuing our efforts into the development of numerical approximation methods for the identification of spatially varying parameters in hybrid models for the transverse vibration of flexible beams with attached appendages. Our investigation has centered upon a Hilbert space-weak/variational formulation of the coupled system of ordinary and partial differential equation which describes the motion of the system. The approximation schemes are spline-based, both with regard to the infinite dimensional state (using a Galerkin approach) and the infinite dimensional admissible parameter space. The convergence of the schemes has been established with numerical testing in progress. Presently the testing has involved the use of synthetic simulation data; however, future plans call for studies involving actual experimental data.

Marsha J. Berger

Previous work has studied the accuracy of an adaptive mesh refinement algorithm for solving the Euler equations for steady two-dimensional transonic flow. Using a global coarse grid and locally uniform fine grid patches only where they are needed (such as at the leading and trailing edges, and at shocks), the same accuracy can be achieved as a grid that is uniformly refined over the entire domain. A multigrid algorithm has been developed by A. Jameson (Princeton University) to accelerate the convergence to steady state. Work is in progress to extend multigrid to non-global fine grids. Preliminary results to date do not obtain the same convergence rate as global grids, although the total work is still reduced since the fine grids are small. This is probably due to the fine/coarse interface conditions. We are experimenting with different formulations to try to improve the convergence characteristics.

Shahid H. Bokhari

Research has continued on the problem of augmenting computer networks. Prior results obtained in collaboration with A. D. Raza (Telephone & Telegraph Dept., Lahore, Pakistan) showed how an N x N mesh could be augmented by adding at most one edge per node in order to perform the shuffle-exchange of size N/2 in 3 time steps. New results have now been obtained that show how a full N item shuffle can be done on this augmented network in 5 time steps. This time is optimal. An ICASE report describing this result is in preparation.

Research on the mapping problem continues in collaboration with M. Ashraf Iqbal (University of Engineering, Lahore, Pakistan). The objectives include developing new heuristics and exploring the possibilities of exact solutions for special cases.

The closely related assignment problem for distributed computer systems is also under study. While this problem is computationally intractable in general, polynomial time solutions have been found for several restricted cases of practical interest. For example, an efficient dynamic programming solution to the problem of optimally pipelining a chain of tasks over an inhomogeneous unidirectional ring of processors has been developed.

Shahid H. Bokhari and Marsha Berger

The possibilities of solving adaptive mesh refinement problems on a multiprocessor system are being explored. A major difficulty is that, no matter how portions of the mesh are initially assigned to processors, refinement will ultimately cause the computational load on the processors to become unbalanced. Attempts at rebalancing are complicated by the need to keep the interprocessor communication overhead at a minimum.

An entirely new method of dissecting the mesh under analysis has been developed. This is based on two-dimensional binary trees and results in an optimal partitioning of the problem. The partioned problem can then be mapped very naturally onto a tree machine. A further dissection of the mesh is done that allows each node of the tree machine to be utilized and ensures that interprocessor communication is minimum. We propose to evaluate the performance of this method on (1) a true tree machine, (2) a shared memory machine (like the NYU Ultra), and (3) a mesh connected machine (such as the NASA Langley FEM).

Dennis W. Brewer

Research is continuing on parameter estimation problems associated with linear evolution equations in infinite-dimensional spaces. A general algorithm based on quasilinearization has been developed and will be numerically tested. The general structure is particularly applicable to delay differential equations and some other forms of functional differential equations. Certain convergence properties of an iterative parameter estimation algorithm have been established. Similar results are now being developed under conditions which will broaden its applicability.

In another effort with J. Steven Gibson, computer programs based on earlier work of Russell Teglas have been developed for estimating parameters in robotic manipulator arms. The programs have been tested with computer generated data, and we expect that real data will soon be available. In a related effort we have also considered the question of choosing input functions which will maximize parameter sensitivity and thereby enhance the accuracy of the estimates.

Stephen F. Davis

Work has continued on the application of upwind finite difference methods to the numerical solution of systems of hyperbolic partial differential equations. Particular emphasis has been focused on methods for computing discontinuous solutions of the Euler equations, i.e., solutions containing shocks.

A recently completed study has shown that it is possible to construct a total variation diminishing (TVD) finite difference scheme which does not require the determination of an "upwind" direction. As a result of this study, a TVD version of the popular MacCormack scheme was constructed and reported on in ICASE Report No. 84-20. At present, a version of this method, applicable to equations written in general non-orthogonal coordinate systems, is being tested by D. Rudy (LaRC). Preliminary results obtained using this scheme compare favorably to results obtained using the more complicated upwind TVD methods.

Currently, this work is proceeding in a number of directions. In particular, we have developed a new limiter for use with the MacCormack TVD scheme and are testing its performance. We are in the process of constructing a central difference TVD artificial viscosity which can be used with implicit schemes, Runge-Kutta schemes, and perhaps spectral methods. We plan to apply these results to both finite volume versions of these algorithms and to equations written in general curvilinear coordinate systems. An ICASE report which describes these developments is in preparation.

A number of researchers have recently noted that diagonally dominant matrices arise from applying Newton-type methods to implicit upwind discretizations of hyperbolic partial differential equations. Since these matrices are amenable to solution by iterative methods, they solve these problems by applying some iterative method a fixed number (usually one or two) of times per Newton step. Although this approach has been successful, we are currently studying an alternative approach whereby, at each Newton step, the linear system is solved to some tolerance which depends on the size of the Newton residual. Results obtained thus far indicate that this approach requires fewer Newton steps to reach convergence, but at this time we do not know that this yields an overall savings in computational effort for realistic problems.

Work is underway (in collaboration with J. P. Drummond, LaRC) on numerical methods for problems involving the interaction of fluid dynamics and finite rate chemistry. At present we are conducting a study to determine which of the many methods that have been developed for stiff ordinary differential equations could be practically adapted to large problems involving partial differential equations.

Stefan Feyock

Work has continued on syntax programming, the use of syntax analysis as a programming and problem-solving tool in a manner analogous to logic programming. The emphasis of this work has shifted to the investigation of optimal methods of specifying the semantics that do the actual computation.

We have investigated the use of a LISP-like language and have in fact found a way of interspersing this notation with the Pascal code that is the "native language" of our syntax programming system. This notation has been used successfully in the construction of a prototype database system based on T. Pratt's concept of H-graphs. More recently we have developed a set of procedures that implements Prolog-like deduction and which can be invoked as part of the semantics. This capability obviates any difficulties arising from syntax programming's lack of backtracking.

George J. Fix

A least squares finite element scheme is being developed in collaboration with C. Cox (Carnegie-Mellon University) for problems of mixed type which arise in treating the transonic flutter problem.

Analysis of mixed type finite element and finite difference scheme is being done with special emphasis on problems from elasticity. Work includes an analysis of stability and accuracy as well as the development of new three-dimensional elements.

Work on iterative methods with special emphasis on spatially varying relaxation parameters is underway. Research is directed toward higher rates of convergence as well as stronger decomposition properties with respect to implementation on parallel computing systems.

Dennis Gannon, Piyush Mehrotra, and John Van Rosendale

There are two principal approaches to programming multiprocessors. Either one can use a language, such as Ada, which contains tasking, synchronization, and communications constructs, or one can rely on restructuring compilers which "hide" the architecture from the programmer. The advantage of restructuring compilers is that they make programming relatively easy and allow portability across architectures. But designing such compilers is a difficult and unsolved research problem.

We are currently exploring several approaches to this problem. One is the use of programming environments based on graphics work stations, which allow users to view program representations (i.e., program flow graphs) and to assist the compiler in transforming and mapping the program to a given architecture. Another approach is to include annotations in the program text, which allow users to suggest appropriate actions at critical points in a program, thus achieving machine specific program restructuring without losing program portability. Current research is directed primarily at the newly developing restructuring compilers for the Blaze parallel language. The first target architecture will be the Flex multiprocessor being installed at Langley Research Center.

James F. Geer

Research is continuing on the following projects:

Comparison of Numerical and Perturbation Techniques to Determine the Aerodynamic Forces on Slender Bodies

This study has involved the comparison of pressure coefficients for some two- and three-dimensional slender bodies of simple geometrical shape using standard numerical (panel) methods and uniform asymptotic methods. (Some preliminary results have been reported in a recent ICASE report.) Work is now continuing to try to determine if some of these results can be "pieced together" to describe the forces acting on more complicated geometrical shapes, such as a typical subsonic aircraft.

The work is being done in collaboration with E. Liu (LaRC) and L. Ting (New York University).

Aircraft Lighting Interaction Problem

This study has involved using slender body theory to derive some approximate analytical solutions to the electromagnetic field problem about some simple geometrical shapes. These shapes include a slender body of revolution (to model the fuselage of an aircraft) and a two-dimensional airfoil. The goal of the project is to help predict the electromagnetic fields inside an aircraft which is either struck by lightning or experiences a nearby strike. Some of the electromagnetic resonances (eigenvalues) and corresponding mode shapes (eigenfunctions) for perfectly conducting slender bodies of revolution have been determined. These results will eventually be used to construct the time history (transient) response of a body excited by an impulsive plane wave.

This work was motivated by several discussions with F. Pitts (LaRC).

Reduced Basis Method Extensions and Applications

This work involves a study of the reduced basis method (a semi-numerical, semi-analytical method which has been used to solve a variety of nonlinear boundary value problems in elasticity) to determine how the method might be

applied to some fluid dynamic and scattering problems. Several model problems are being studied both to understand the method better and to determine its applicability. At present, these model problems include some two-point boundary value problems with boundary layers and some simple exterior scattering problems.

The is work is being done with C. Anderson (College of William and Mary).

Nonlinear Oscillations Using Symbolic Computation

This study involves the use of the symbolic computation system MACSYMA and perturbation methods to investigate some free and forced nonlinear oscillations. The method of multiple time scales has been implemented using MACSYMA and applied to the problem of determining the transient responses of the van der Pol and Duffing oscillators when the applied force consists of a sum of periodic terms with different frequencies. The conditions under which the system will experience frequency entrainment are being investigated.

J. Steven Gibson and I. Gary Rosen

We are developing numerical approximation schemes for the closed loop solution (determination of feedback gains) of infinite dimensional discrete time linear quadratic regulator problems. We are investigating spline based methods for both the finite and infinite time horizon problems. The methods yield approximations to the feedback gain operators in the form of solutions to standard finite dimensional Riccati type difference (for the finite time horizon problem) or algebraic (for the infinite time horizon problem) equations which can be computed using well known techniques and readily available software. Theoretical convergence arguments have been completed with numerical studies currently underway. Examples under consideration include heat conduction with boundary control, vibration damping for flexible structures, and the control of hereditary or delay systems.

J. Steven Gibson and Dennis Brewer

We have developed software for identifying parameters in robotic manipulators. The programs use numerical integration of nonlinear differential equations, nonlinear optimization algorithms, and large matrix computations. So far, the programs have been tested on simulated data only. We expect to get laboratory data from the NASA robotics group. The software needs further development to accommodate noisy data and more complex manipulator dynamics. Also, input optimization for parameters should be a fruitful area for future research.

Chester E. Grosch

A series of calculations (in collaboration with T. Gatski, LaRC) has been initiated whose purpose is to examine the effect of embedded cavities on separating flow. Also, large scale turbulent structures are being modeled in these flows by using Stuart vortices.

A modification of the two-dimensional Navier-Stokes code of Gatski, Grosch, and Rose is being used to calculate the impulsive start-up of a slender elliptic cylinder. The objective of this work is to study the time evolution of the flow past this slender, blunt-nosed body -- particularly the development in time of the separation region at the rear of the body, as a function of slenderness ratio and Reynolds number. The results of the calculation will be compared with the predictions of classical boundary layer theory and with those of triple-deck theory. We also plan to calculate the flow due to the interaction of vortical disturbances in the free stream ahead of the body with the blunt leading edge.

Finally, in collaboration with T. Gatski and M. E. Rose, a three-dimensional Navier-Stokes code in vorticity and velocity variables using compact schemes is being developed. This code is now being tested by calculating the time evolution of the Taylor-Green vortex.

Max D. Gunzburger and Roy A. Nicolaides

Numerical studies of incompressible separated flows are underway using codes implementing finite element methods for the Navier-Stokes equations. For the most part, two types of problems are considered. The first is boundary layer flows over irregular obstacles such as ramps, curved ramps, steps, cavities, and combinations of these. Here a careful study of the effects of outflow boundary conditions has been carried out. The purpose of these calculations is to study the effects of the various obstacles on separation and, hopefully, to predict the appearance of transition regions. The calculations will be compared with experiments being conducted by B. Holmes (LaRC). The second project is the calculation of flows around airfoils at high angles of attack. A study of turbulence models which may be incorporated into the finite element codes in an efficient manner has been carried out. Present efforts are being directed at developing codes to incorporate these models. In addition to the above personnel, the project has involved students at Carnegie-Mellon University.

Subramaniya I. Hariharan

A two-dimensional model is being considered (in collabortion with H. Lester, LaRC) to study the nonlinear behavior of acoustic waves. A code has been developed using a fourth-order spatial accurate scheme with operator splitting techniques. Currently we are examining possibilities of alternate methods that do not incorporate operator splitting. Moreover, implementation of a wave envelope technique for this nonlinear problem is planned.

A survey study of absorbing boundary conditions for elliptic and hyperbolic problems is underway. Currently there are various types of absorbing boundary conditions used to simulate infinite regions computationally. They are either local or nonlocal differential or integrodifferential types. This study will update information on available boundary conditions, including our treatment.

A computational study has begun on the study of nonlinear acoustic wave propagation in atmosphere. It is known in linear theory that the amplitude of

the measured waves is directly proportional to the source amplitude. However, in reality saturation occurs, and the experiments show that this linear behavior does not occur. The particular emphasis in this research is to study long-range low-frequency behavior of acoustic waves. The first stage of this work will consist of an asymptotic study that can be compared with computational methods.

Murshed Hossain

High Reynolds number turbulent shear flows have a great range of temporal and spatial scales. It is almost impossible to simulate them on presently available or projected future computers. Large eddy simulation provides a viable alternative. This is based on the assumption that the velocity field can be divided into large and small eddies, and the motions of the large eddies are not sensitive to the details of the small eddies (smaller than the grid size of a sufficiently fine mesh). The small eddies are assumed to have a universal character (there is evidence to this effect) and are hence amenable to modelling. This is called the subgrid scale modelling. The purpose of the present study is to estimate the eddy-viscosity by systematic removal of small scales from the dynamics using a renormalization technique. This work is in progress in collaboration with G. Vahala and Y. Zhou (William and Mary).

It is well known that free stream turbulence, entropy disturbance in tunnel acoustic field, triggers unstable waves in a boundary layer. But the mechanism by which an external disturbance environment enters the boundary layer (and the nature of its signature in the perturbed flow) is not well understood. The object of the present research effort is to perform controlled numerical experiments to get some insight into these mechanisms. With this purpose in view, we are constructing an algorithm for solving three-dimensional Navier-Stokes equations. Among the physical problems we intend to study are: response of a boundary layer to a stationary or moving acoustic wave and convected array of counter rotating vortices. So far we have an explicit code validated against the linear theory.

M. Yousuff Hussaini

The program of research in hydrodynamic stability and transition to turbulence is continuing. A systematic study of the effects of laminar flow control techniques (such as heating and suction) on the transition process of a Blasius boundary layer is being carried out. The problem of receptivity is being formulated within the framework of linearized Navier-Stokes equations. Some of the classical problems such as Rayleigh-Benard convection and Taylor-Couette flow will be re-examined computationally in view of the increased resolution possible on LaRC computers.

Certain physical mechanisms at play in shock wave/boundary layer interaction problems were identified, and three of these were studied under ideal isolated conditions (AIAA J. Vol. 22, 1984, pp. 13-21). In the present study, the effect of shock oscillations on turbulence enhancement is investigated. The model consists of an oscillating wedge in supersonic flow. The flow situation is assumed to be governed by linearized Euler equations, and analytical/numerical solutions are obtained.

Dennis Bushnell (LaRC) proposed that the shock-induced separation of flow could be prevented by placing a control surface in the outer part of the boundary layer to bounce off the shock. This has been confirmed by the numerical solution of the two-dimensional compressible Navier-Stokes equations. Numerical experiments are continuing to determine the optimal positioning of the control surface.

We plan to study phenomenological aspects of quasi-steady three dimensional flow separation by simulating the flow past a circular cylinder standing normal to a flat plate. Mach numbers for these numerical simulations will lie in the subsonic to supersonic range. We will attempt to throw light on the relevance of Maskell's envelope approach and Lighthill's asymptote arguments to the flow conditions in the vicinity of separation and reattachment lines. Furthermore, the dependence of the vortical structures on Reynolds number will be investigated.

In another study, the dynamics of an eddy suddenly interjected in a supersonic stream is investigated. Although this study is confined to two-dimensional unsteady Navier-Stokes equations, it has relevance to the eddy shocklet phenomena in a sense observed in supersonic turbulent jets.

Large eddy break up (LEBU) experiments of turbulent flows report significant reduction in local skin friction and net drag for some laboratory conditions. It is important to understand the basic physics of extremely complex flow/surface interactions. As an initial approximation, we propose to study a more tractable problem which incorporates the important features of the flow. With this viewpoint, we will consider the two-dimensional laminar flow over a flat plate with a LEBU device (consisting of a single or multiple rectangular plate elements). The large eddy is modelled by a two-dimensional compressible vortex convected with the flow. The relevant compressible Navier-Stokes equations are solved with proper inflow and outflow conditions for free stream Mach numbers ranging from subsonic to supersonic conditions.

Kazufumi Ito

Research on delay differential equations is continuing. The characteristic equation for the open and closed loop eigenvalues of the approximate system via the Legendre-Tau method has been derived (see ICASE Report No. 84-42). It leads to an algorithm to construct a finite-order compensator for retarded systems. A question of the regularity of optimal feedback gain operators for the linear quadratic regulator problem is being investigated. The regularity question is important for finding a rate of convergence of numerical methods for approximating the feedback gain operators.

Research has continued in collaboration with H. T. Banks on the development of computational algorithms for feedback synthesis for systems governed by partial differential equations. Our research effort is focused on control of a beam-like structure with tip body. The study also involves the development of efficient algorithms to solve Riccati equations.

David A. Kopriva

A method has been devised for patching together two (or more) Chebyshev grids so that increased resolution can be obtained in different parts of a hyperbolic flow. The method is simple and efficient for hyperbolic systems written in nonconservative form. Numerical tests indicate that it is stable and that spectral accuracy is retained. The extension of the method to treat the conservative form of the equations is being studied.

Development of a spectral collocation code to compute two-dimensional shock-fitted flows over bodies is continuing. Shock-fitting and shock detection algorithms have been tested for both the two-dimensional code and a one-dimensional nozzle problem. The mesh patching strategy above will be incorporated into the final code. The program will be used to perform benchmark calculations against which finite difference solutions will be compared.

William D. Lakin and M. Y. Hussaini

This work considers a Blasius boundary-value problem with inhomogeneous lower boundary conditions $f(0) = 0$ and $f'(0) = -\lambda$ with λ strictly positive. The Crocco variable formulation of this problem has a key term which changes sign in the interval of interest. It is shown that solutions of the boundary-value problem do not exist for values of λ larger than a positive critical value λ^*. The existence of solutions is proved for $0 < \lambda \leq \lambda^*$ by considering an equivalent initial value problem. However, for $0 < \lambda < \lambda^*$, solutions of the boundary-value problem are found to be nonunique. Physically, this nonuniqueness is related to multiple values of the skin friction.

Liviu Lustman

A study of compact schemes, as applied to unsteady elasticity problems, is underway in collaboration with M. Rose. These numerical methods are expected

to be applicable to vibrational structure analysis for complex shapes and composite materials. As compact schemes always result in implicit equations, special attention will be given to efficient iterative methods of solution.

In collaboration with D. Ebin (S.U.N.Y. at Stony Brook) a computational algorithm for periodic incompressible inviscid flow is being analyzed. Particle paths are tracked by explicitly using vorticity convection (in two dimensions) and vorticity evolution in three-dimensional flow. Work is in progress in improving the accuracy of the method, in particular to obtain better volume conservation.

Research into applications of spectral methods is continuing in the area of numerical boundary conditions and boundary layer resolution.

Mala Mehrotra and John Van Rosendale

Tree searching is a fundamental technique in computer science, having applications in combinatorial optimization, artificial intelligence, robotics, numerical analysis, and computer game-playing. The research here focuses on the problem of mapping tree searching algorithms to loosely coupled multi-microprocessor architectures. Though there is clearly substantial parallelism in most tree searching problems, this parallelism can be difficult to exploit on highly parallel, loosely coupled machines. Our work is directed at discovering heuristically based adaptive load distribution algorithms which can distribute the load nearly uniformly over the processors, without incurring excessive communications costs. Simulation results obtained so far are encouraging, suggesting that relatively simple heuristic strategies can map tree searching algorithms to multiprocessor architectures quite well. There is also some hope that the results obtained will shed light on the subtle problem of designing restructuring compilers to map applicative programming languages onto loosely coupled parallel architectures.

Piyush Mehrotra and John Van Rosendale

One of the central issues in parallel computing research is the design of software environments for parallel architectures. For a variety of reasons, it seems far easier to program sequential computers than parallel machines. In particular, designing the concurrent programs having multiple threads of control flow needed for MIMD multiprocessors has proven quite subtle.

One of the first questions that needs to be asked is whether the difficulty experienced in programming multiprocessors is inherent in parallel execution or is simply a reflection of the inadequacy of current software tools. Put differently, one can ask whether it is possible to design programming environments which allow one to write correct and efficient parallel programs as easily as one currently writes sequential programs.

In order to address this question, the authors have defined a Pascal-like programming language, Blaze, designed for scientific computing. Though Blaze resembles conventional programming languages, it relies on applicative procedure calls, allowing restructuring compilers automatically to extract coarse grained "multi-tasking" parallelism. Implementation of this language is currently underway, and experiments with alternate approaches to parallel run-time environments will be reported soon.

Vijay K. Naik and Joel Saltz

There are numerous problems involving time dependent partial differential equations for which obtaining a relatively accurate solution can be an expensive, time consuming process. In many algorithms for the solution of such problems, it is necessary to solve a sequence of linear equations involving the same coefficient matrix but with different "right-hand sides," which are dependent on the solutions of the equations corresponding to the earlier time steps in the sequence.

Our research is aimed at discovering how to solve these problems efficiently on multiprocessors. One of the factors that reduces the efficiency of multi-processor algorithms and limits the number of processors that may be usefully employed is the amount and the timing of communication

that must take place between problem partitions that run on different processors. Iterative algorithms that involve partitioning the domain of the partial differential equation, with each partition sweeping over several time steps, reduce synchronization requirements and allow overlap of computation and communication. The performance implications of this communication and computation overlap vary with the architecture of the multi-processor system and are being analyzed.

Vijay Naik and John Van Rosendale

An examination of numerical programs indicates that most of the available parallelism is concentrated at the loop level, rather than at the procedure level. One of the recent innovations in vector computers is "scatter-gather" hardware, which allows general permutations of data, thereby broadening the class of loops which can be performed efficiently. But for many types of loops the ability to permute data isn't sufficient. For example, the computation of the product of a sparse matrix and a vector cannot be done easily on a vector computer, even if scatter-gather hardware is available.

A natural generalization of permutation is the "replace-add" or "fetch-add" operation of the NYU Ultracomputer. This operation allows many processors in a multiprocessor system to access and update semaphores in parallel. Many numerical problems, including sparse matrix problems and particle-in-cell plasma simulations, require exactly this type of operation. But "replace-add" hardware is probably not the complete answer since, for one thing, the size of problem which can be solved is limited by the size of the "replace-add" network available. As an alternative, we are studying the use of single-stage multi-microprocessor networks which can be used to emulate a "replace-add" network of arbitrary size. Simulation results obtained indicate that this approach reduces or eliminates load balancing problems and that the use of a single-stage network used in this way may be more attractive than the use of Log-stage networks, such as the Omega network.

Robert Noonan

The last few years has seen the development of a number of table-driven code generation schemes. To date, all of the experience with these systems has been for conventional, single processor machines. This research will investigate the applicability of these schemes to flexible architecture machines employing many processors. Possible target machines include the FLEX and the Intel 432.

Initial work has centered around porting the Mystro system developed at the College of William and Mary to the VAX. A minor topic is the investigation of various syntactic error recovery techniques for use in the implementation of BLAZE.

Merrell L. Patrick and Daniel A. Reed

A model of a general class of asynchronous, iterative solution methods for random, sparse linear systems has been developed. In the model, the system is solved by creating several cooperating tasks that each compute a portion of the solution vector. A data transfer model predicting both the probability that data must be transferred between two tasks and the amount of data to be transferred is included. This model is used to derive an execution time model for predicting parallel execution time and an optimal number of tasks given the dimension and sparsity of the coeffient matrix and the costs of computation, sychronization, and communication.

The suitability of different parallel architectures for solving randomly sparse linear systems is being considered. Based on the complexity of task scheduling, a model of one parallel architecture, based on a broadcast bus, has been developed and analyzed. Data flow architectures are currently being studied.

Timothy N. Phillips

In joint work with M. Y. Hussaini and T. Zang (LaRC), investigations have been made comparing various relaxation schemes within the context of the spectral multigrid method. Progress has been made on preconditioning the matrices which arise from problems with periodic boundary conditions in one direction. The results will appear in a future paper. Similar ideas are being pursued with M. M. Hafez (Computer Dynamics, Inc.) in connection with finite difference discretizations of periodic problems. The details of this work will appear in a forthcoming ICASE report.

Work with M. M. Hafez to extend the modified least squares method (ICASE Report No. 84-16) to treat the transonic small disturbance equation is in progress. This work will be reported in a paper on the numerical solution of transonic flow problems.

Robert K. Powers

Research on deriving Chandrasekhar equations for infinite dimensional linear quadratic regulator (LQR) problems is continuing in conjunction with K. Ito. We have derived Chandrasekhar equations for LQR problems with bounded inputs and bounded outputs. Extensions to the unbounded case are being sought. Numerical solutions obtained for the LQR problem via the Chandrasekhar equations have been observed to be computationally efficient for delay differential systems. The features of the Chandrasekhar equations which allow efficient computations in the delay systems also appear to be present in the boundary control and observation of general distributed parameter systems. Investigations into the numerical aspects of this are continuing.

Research is also continuing (in collaboration with J. Burns, VPI & SU) on hereditary models for active flutter control of a thin airfoil. Chandrasekhar equations are being solved to obtain solutions to the associated LQR problem in a computationally efficient manner (see ICASE Report No. 84-50).

Terrence W. Pratt

The programming of scientific and engineering problems for MIMD parallel computers is hampered by the lack of suitable programming environments. In particular, it is desirable to experiment with programming complete codes involving parallel algorithms. Performance comparisons for the same programs running on a variety of parallel machine architectures are particularly desirable, given the range of architectures that are presently under construction or proposed.

A project is underway at ICASE to construct an appropriate parallel programming environment. The base programming language is Fortran 77, extended to include parallel constructs. Initially the system has been implemented under UNIX on the ICASE VAX-750, with the intention of moving the implementation to a parallel machine when one becomes available at LaRC. The system is known by the acronym PISCES (Parallel Implementation of Scientific Computing EnvironmentS).

The Pisces design is based on a formally specified "virtual machine." The virtual machine provides parallel operations of five different "granularities," ranging from vector operations (instruction-level parallelism) to asynchronous tasks that communicate through messages. The new statements and declarations in Pisces Fortran provide access to the various capabilities of the Pisces virtual machine. Each implementation of the virtual machine on a particular parallel architecture is expected to directly provide parallel operation at one or more of the granularity levels and simulate the other levels with ordinary sequential execution. Thus the same program may be run on several different implementations without major change in the code, but with potentially major performance differences due to differences in the underlying parallel architecture. By using Fortran as the base sequential language, the sequential parts of many programs may be carried over intact into the parallel environment without reprogramming.

Milton E. Rose

If an elastic body is in equilibrium due to boundary constraints, each volume element will be in isolated equilibrium due to traction forces and displacements on the surface of the element and the traction force acting across any face common to two neighboring elements will vanish. Also, the potential energy in any element will equal the work done by the traction forces and displacements on the boundary of the element and the total potential energy can be obtained by summation. For an element in isolated equilibrium, the surface traction is a simple functional of the surface displacements and the work on the element is a quadratic functional of the surface displacements. As a result, the minimum potential energy of the body also minimizes the work due to displacement constraints on the surfaces of volume elements.

These observations form the basis for a nonconforming finite element approximation method which, for brick elements, yields compact finite difference equations described earlier (ICASE Report No. 84-2).

With L. Lustman, applications of the general method to treat composite materials are being studied using a homogenization technique suggested by the method. Fast iterative solution methods also appear likely. An extension to treat the dynamic problem has been developed and is being studied.

Nancy E. Shoemaker

The ICASE VAX system continues to evolve. The purchase of a half dozen new terminals allowed us to reach the goal of one terminal in each scientist's office. ICASE joined CSNET in May, and a number of people have begun to take advantage of access to Arpanet mailing lists.

A major disappointment during the period was a delay in completing the licensing arrangements for 4.2BSD UNIX. It is hoped that once 4.2 is in place, work can begin on integrating the ICASE VAX with the Langley local area network. In the meantime, with J. Stanley, we improved the remote job entry connection between the VAX and the ACD complex. With this enhanced system in place, we continue to investigate the use of UNIX tools to create commands that would request services of the ACD complex.

A survey of the staff in the summer of 1984 revealed a perceived need for improved graphics and a better integrated document preparation system. First steps in the direction of meeting these needs were ordering of a Ridge/32 workstation and arranging an evaluation of the MUSE word processing system.

In April 1984, a Langley UNIX special interest group was formed. About 80 people expressed an interest in UNIX and about 25 attended the first SIG meeting. During the six months covered by this report, the interest in UNIX at Langley has grown dramatically. It is hoped that the UNIX-SIG can be used during the next six months to facilitate a center-wide approach to UNIX training. In the meantime, many people have been issued "training accounts" on the ICASE VAX, and we have helped in the installation of a few UNIX workstations in various branches. During the summer of 1984, A. Holloway rewrote the introductory material distributed to new users of the ICASE VAX and these 'UNIX for Intermediates' papers will be made available to other UNIX installations at Langley.

Robert G. Voigt

Preparation of an extensive review of the state of parallel numerical algorithms for partial differential equations has continued in collaboration with J. Ortega. The work will include an overview of the architectural issues of parallel and vector computers that influence algorithm performance. Both direct and iterative techniques are considered along with an indication of the kinds of applications that have been addressed. An extensive bibliography will also be included.

Another effort with J. Oliger involves studying the nature of high performance scientific computing. This involves many stages and disciplines: problem definition--engineering and the sciences; mathematical formulation--engineering, the sciences, and applied mathematics; discretization and algorithm formulation--numerical and algorithmic analysis; algorithm description and implementation--computer languages and systems; execution of the calculation--computer hardware; and interpretation of the results--data base management, graphics, etc. A central theme is that

synergism between these phases and disciplines is necessary to produce a modern computational environment; that is, one which can be easily used, employs modern technology, exploits parallelism to a high degree, and utilizes sophisticated, efficient algorithms. Several other researchers have been asked to write on various aspects of the synergistic process, and the result is intended as a volume in the SIAM Frontiers of Applied Mathematics series.

REPORTS AND ABSTRACTS

April 1, 1984 through September 30, 1984

Ito, Kazufumi: *An iterative method for indefinite systems of linear equations.* ICASE Report No. 84-13, April 2, 1984, 21 pages. Submitted to SIAM J. Numer. Anal.

An iterative method for solving nonsymmetric indefinite linear systems is proposed. The method involves the successive use of a modified version of the conjugate residual method. A numerical example is given to illustrate the method.

Adams, Loyce M. and Harry F. Jordan: *Is SOR color-blind?* ICASE Report No. 84-14, May 21, 1984, 39 pages. Submitted to SIAM J. Sci. Statist. Comput.

The work of Young [1971] showed that the Red/Black ordering and the natural rowwise ordering of matrices with Property A, such as those arising from the 5-point discretization of Poisson's equation, lead to SOR iteration matrices with identical eigenvalues. With the advent of parallel computers, multi-color point SOR schemes have been proposed for more complicated stencils on two-dimensional rectangular grids; see Adams and Ortega [1982] for example, but to our knowledge, no theory has been provided for the rate of convergence of these methods relative to that of the natural rowwise scheme.

New results show that certain matrices may be reordered so the resulting multi-color SOR matrix has the same eigenvalues as that for the original ordering. In addition, for a wide range of stencils, we show how to choose multi-color orderings so the multi-color SOR matrices have the same eigenvalues as the natural rowwise SOR matrix. The strategy for obtaining these results is based on "data flow" concepts and can be used to reach Young's conclusions above for the 5-point stencil.

The importance of these results is threefold. Firstly, a constructive and easy means of finding these multi-colorings is a direct consequence of the theory; secondly, multi-color SOR methods can be found that have the same rate of convergence as the natural rowwise SOR method for a wide range of stencils used to discretize partial differential equations; and thirdly, these multi-color SOR methods can be efficiently implemented on a wide class of parallel computers.

Gatski, Thomas B. and Chester E. Grosch: *A numerical study of the two- and three-dimensional unsteady Navier-Stokes equations in velocity-vorticity variables using compact difference schemes.* ICASE Report No. 84-15, May 22, 1984, 20 pages. Proc. of Ninth International Conference on Numerical Methods in Fluid Dynamics, Lecture Notes in Physics, Springer-Verlag, Saclay, France, June 23-29, 1984.

A compact finite-difference approximation to the unsteady Navier-Stokes equations in velocity-vorticity variables is used to numerically simulate a number of flows. These include two-dimensional laminar flow of a vortex evolving over a flat plate with an embedded cavity, the unsteady flow over an elliptic cylinder, and aspects of the transient dynamics of the flow over a rearward facing step. The methodology required to extend the two-dimensional formulation to three-dimensions is presented.

Hafez, Mohamed, M. and Phillips, Timothy N.: *A modified least squares formulation for a system of first-order equations.* ICASE Report No. 84-16, May 29, 1984, 21 pages. Submitted to IMACS J. Numer. Math.

Second-order equations in terms of auxiliary variables similar to potential and stream functions are obtained by applying a weighted least squares formulation to a first-order system. The additional boundary conditions which are necessary to solve the higher order equations are determined and numerical results are presented for the Cauchy-Riemann equations.

Hall, Phillip: *The Gortler vortex instability mechanism in three-dimensional boundary layers.* ICASE Report No. 84-17, June 6, 1984, 37 pages. Submitted to the Royal Society of London.

It is well known that the two-dimensional boundary layer on a concave wall is centrifugally unstable with respect to vortices aligned with the basic flow for sufficiently high values of the Gortler number. However, in most situations of practical interest the basic flow is three-dimensional, and previous theoretical investigations do not apply. In this paper the linear stability of the flow over an infinitely long swept wall of variable curvature is considered. If there is no pressure gradient in the boundary layer, it is shown that the instability problem can always be related to an equivalent two-dimensional calculation. However, in general, this is not the case and even for small values of the crossflow velocity field dramatic differences between the two and three-dimensional problems emerge. In particular, it is shown that when the relative size of the crossflow and chordwise flow is $O(R^{-1/2})$, where R is the Reynolds number of the flow, the most unstable mode is time-dependent. When the size of the crossflow is further increased, the vortices in the neutral location have their axes locally perpendicular to

the vortex lines of the basic flow. In this regime the eigenfunctions associated with the instability become essentially 'centre modes' of the Orr-Sommerfeld equation destabilized by centrifugal effects. The critical Gortler number for such modes can be predicted by a large wavenumber asymptotic analysis; the results suggest that for order unity values of the ratio of the crossflow and chordwise velocity fields, the Gortler instability mechanism is almost certainly not operational.

Gozani, J., Nachshon, A., and Turkel, E.: *Conjugate gradient coupled with multigrid for an indefinite problem.* ICASE Report No. 84-18, June 7, 1984, 11 pages. Submitted to Proc. of Third IMACS International Symposium on Computer Methods for Partial Differential Equations, Lehigh University, Bethlehem, PA, June 20-22, 1984.

An iterative algorithm for the Helmholtz equation is presented. This scheme is based on the preconditioned conjugate gradient method for the normal equations. The preconditioning is one cycle of a multigrid method for the discrete Laplacian. The smoothing algorithm is red-black Gauss-Seidel and is constructed so it is a symmetric operator. The total number of iterations needed by the algorithm is independent of h. By varying the number of grids, the number of iterations depends only weakly on k when k^3h^2 is constant. Comparisons with a SSOR preconditioner are presented.

Malik, M. R., Zang, T. A., and Hussaini, M. Y.: *A spectral collocation method for the Navier-Stokes equations.* ICASE Report No. 84-19, June 8, 1984, 48 pages. Submitted to J. Comput. Phys.

A Fourier-Chebyshev spectral method for the incompressible Navier-Stokes equations is described. It is applicable to a variety of problems including some with fluid properties which vary strongly both in the normal direction and in time. In this fully spectral algorithm, a preconditioned iterative technique is used for solving the implicit equations arising from semi-implicit treatment of pressure, mean advection and vertical diffusion terms. The algorithm is tested by applying it to hydrodynamic stability problems in channel flow and in external boundary layers with both constant and variable viscosity.

Davis, Stephen F.: *TVD finite difference schemes and artificial viscosity.* ICASE Report No. 84-20, June 11, 1984, 35 pages. To appear in SIAM J. Sci. Statis. Comput.

In this paper we show that the total variation diminishing (TVD) finite difference scheme which was analysed by Sweby [6] can be interpreted as a Lax-Wendroff scheme plus an upwind weighted artificial dissipation term. We then show that if we choose a particular flux limiter and remove the requirement for upwind weighting, we obtain an artificial dissipation term which is based on the theory of TVD schemes, which does not contain any problem dependent parameters and which can be added to existing MacCormack method codes. Finally, we conduct numerical experiments to examine the performance of this new method.

Canuto, C., Hariharan, S. I., and Lustman, L.: *Spectral methods for exterior elliptic problems.* ICASE Report No. 84-21, June 12, 1984, 33 pages. To appear in Numer. Math.

This paper deals with spectral approximations for exterior elliptic problems in two dimensions. As in the conventional finite difference or finite element methods, it is found that the accuracy of the numerical solutions is limited by the order of the numerical farfield conditions. We introduce a spectral boundary treatment at infinity, which is compatible with the "infinite order" interior spectral scheme. Computational results are presented to demonstrate the spectral accuracy attainable. Although we deal with a simple Laplace problem throughout the paper, our analysis covers more complex and general cases.

Graif, E. and K. Kunisch: *Parameter estimation for the Euler-Bernoulli-beam.* ICASE Report No. 84-22, June 20, 1984, 43 pages. Submitted to IEEE Trans. Auto. Control.

An approximation involving cubic spline functions for parameter estimation problems in the Euler-Bernoulli-beam equation (phrased as an optimization problem with respect to the parameters) is described and convergence is proved. The resulting algorithm was implemented and several of the test examples are documented. It is observed that the use of penalty terms in the cost functional can improve the rate of convergence.

Rosen, I. Gary: *A numerical scheme for the identification of hybrid systems describing the vibration of flexible beams with tip bodies.* ICASE Report No. 84-23, June 21, 1984, 36 pages. Submitted to J. Math. Anal. Appl.

A cubic spline based Galerkin-like method is developed for the identification of a class of hybrid systems which describe the transverse vibration of flexible beams with attached tip bodies. The identification problem is formulated as a least squares fit to data subject to the system dynamics given by a coupled system of ordinary and partial differential equations recast as an abstract evolution equation (AEE) in an appropriate infinite dimensional Hilbert space. Projecting the AEE into spline-based subspaces leads naturally to a sequence of approximating finite dimensional identification problems. The solutions to these problems are shown to exist, are relatively easily computed, and are shown to, in some sense, converge to solutions to the original identification problem. Numerical results for a variety of examples are discussed.

Banks, H. Thomas and Katherine A. Murphy: *Estimation of coefficients and boundary parameters in hyperbolic systems.* ICASE Report No. 84-24, June 21, 1984, 52 pages. Submitted to SIAM J. Control Optim.

We consider semi-discrete Galerkin approximation schemes in connection with inverse problems for the estimation of spatially varying coefficients and boundary condition parameters in second order hyperbolic systems typical of those arising in 1-D surface seismic problems. Spline based algorithms are proposed for which theoretical convergence results along with a representative sample of numerical findings are given.

Vardi, A.: *A new minmax problem.* ICASE Report No. 84-25, June 26, 1984, 35 pages. Submitted to J. Optim., Theory and Appl.

The paper deals with the minimax problem $\min_{x \in \mathbb{R}^n} \max_{i=1,\ldots,m} f_i(x)$.

We work with its equivalent representation $\min t$ s.t. $f_i(x) - t \leq 0$ for all i. For this problem we design a new active set strategy in which there are three types of functions: active, semi-active, and non-active. This technique will help in preventing zigzagging which often occurs when an active set strategy is used. Some of the inequality constraints are handled with slack variables. Also a trust region strategy is used in which at each iteration there is a sphere around the current point in which the local approximation of the function is trusted. The algorithm suggested in the paper was implemented into a successful computer program. Numerical results are provided.

Banks, H. T. and I. G. Rosen: *Approximation techniques for parameter estimation and feedback control for distributed models of large flexible structures.* ICASE Report No. 84-26, June 22, 1984, 19 pages. Submitted to Workshop on Identification and Control of Flexible Space Structures, G. Rodgriguez (ed.), San Diego, CA, June 4-6, 1984.

We discuss approximation ideas that can be used in parameter estimation and feedback control for Euler-Bernoulli models of elastic systems. Focusing on parameter estimation problems, we outline how one can obtain convergence results for cubic spline-based schemes for hybrid models involving an elastic cantilevered beam with tip mass and base acceleration. Sample numerical findings are also presented.

Turkel, E. and Bram van Leer: *Flux vector splitting and Runge-Kutta methods for the Euler equations.* ICASE Report No. 84-27, June 23, 2984, 9 pages. Proc. of Ninth International Conference on Numerical Methods in Fluid Dynamics, Lecture Notes in Physics, Springer-Verlag, Saclay, France, June 23-29, 1984.

Runge-Kutta schemes have been used as a method of solving the Euler equations exterior to an airfoil. In the past this has been coupled with central differences and an artificial viscosity in space. In this study we couple the Runge-Kutta time stepping scheme with an upwinded space approximation based on flux-vector splitting. Several acceleration techniques are also considered including a local time step, residual smoothing and multigrid.

Turkel, Eli: *Fast solutions to the steady state compressible and incompressible fluid dynamics equations.* ICASE Report No. 84-28, June 23, 1984, 8 pages. Proc. of Ninth International Conference on Numerical Methods in Fluid Dynamics, Lecture Notes in Physics, Springer-Verlag, Saclay, France, June 23-29, 1984.

It is well known that for low speed flows the use of the compressible fluid dynamic equations is inefficient. The use of an explicit scheme requires Δt to be bounded by $1/c$. However, the physical parameters change over time scales of order $1/u$ which is much larger. Hence, it is not appropriate to use explicit schemes for very subsonic flows. Implicit schemes are hard to vectorize and frequently do not converge quickly for very subsonic flows. We shall demonstrate that if one is only interested in the steady state then a minor change to an existing code can greatly increase the efficiency of an explicit method. Even when using an implicit method the proposed changes increase the efficiency of the scheme. We shall first consider the Euler equations for low speed flows and then incompressible flows. We then indicate how to generalize the method to include viscous effects. We also show how to accelerate supersonic flow by essentially decoupling the equations.

Gottlieb, D.: *Spectral methods for compressible flow problems.* ICASE Report No. 84-29, June 21, 1984, 20 pages. Proc. of Ninth International Conference on Numerical Methods in Fluid Dynamics, Lecture Notes in Physics, Springer-Verlag, Saclay, France, June 23-29, 1984.

In this article we review recent results concerning numerical simulation of shock waves using spectral methods. We discuss shock fitting techniques as well as shock capturing techniques with finite difference artificial viscosity. We also discuss the notion of the information contained in the numerical results obtained by spectral methods and show how this information can be recovered.

Ipsen, Ilse C. F.: *Singular value decomposition with systolic arrays.* ICASE Report No. 84-30, July 9, 1984, 22 pages. Proc. of the Society of Photo-Optical Engineers, San Diego, CA, August 19-24, 1984.

Systolic arrays for determining the Singular Value Decomposition of a $m \times n$, $m \geq n$, matrix A of bandwidth w are presented. After A has been reduced to bidiagonal form B by means of Givens plane rotations, the singular values of B are computed by the Golub-Reinsch iteration. The products of plane rotations form the matrices of left and right singular vectors.

Assuming each processor can compute or apply a plane rotation, $O(wn)$ processors accomplish the reduction to bidiagonal form in $O(np)$ steps, where p is the number of superdiagonals. A constant number of processors can then determine each singular value in about $6n$ steps. The singular vectors are computed by rerouting the rotations through the arrays used for the reduction to bidiagonal form, or else "along the way" by employing another rectangular array of $O(wm)$ processors.

Ito, Kazufumi and Russell Teglas: *Legendre-Tau approximation for functional differential equations. Part II: The linear quadratic control problem.* ICASE Report No. 84-31, July 6, 1984, 65 pages. Submitted to SIAM J. Control Optim.

The numerical scheme based on the Legendre-tau approximation is proposed to approximate the feedback solution to the linear quadratic optimal control problem for hereditary differential systems. The convergence property is established using Trotter ideas. The method yields very good approximations at low orders and provides an approximation technique for computing closed-loop eigenvalues of the feedback system. A comparison with existing methods (based on "averaging" and "spline" approximations) is made.

Turkel, Eli: *Acceleration to a steady state for the Euler equations.* ICASE Report No. 84-32, July 7, 1984, 45 pages. Proc. INRIA Euler Workshop (SIAM), Rocquencourt, France, December 7-9, 1983.

A multi-stage Runge-Kutta method is analyzed for solving the Euler equations exterior to an airfoil. Highly subsonic, transonic and supersonic flows are evaluated. Various techniques for accelerating the convergence to a steady state are introduced and analyzed.

Bokhari, Shahid H. and A. D. Raza: *Augmenting computer networks.* ICASE Report No. 84-33, July 8, 1984, 16 pages. Proc. 1984 International Conference on Parallel Processing, Robert M. Keller, ed., IEEE Computer Society Press, Silver Spring, MD, 1984, pp. 338-345.

Three methods of augmenting computer networks by adding at most one link per processor are discussed.

1. A tree of N nodes may be augmented such that the resulting graph has diameter no greater than $4 \log_2((N+2)/3) - 2$. This $O(N^3)$ algorithm can be applied to any spanning tree of a connected graph to reduce the diameter of that graph to $O(\log N)$.
2. Given a binary tree T and a chain C of N nodes each, C may be augmented to produce C' so that T is a subgraph of C'. This algorithm is $O(N)$ and may be used to produce augmented chains or rings that have diameter no greater than $2 \log_2((N+2)/3)$ and are planar.
3. Any rectangular two-dimensional 4 (8) nearest neighbor array of size $N = 2^k$ may be augmented so that it can emulate a single step shuffle-exchange network of size $N/2$ in $3(t)$ time steps.

Reed, Daniel A. and Merrell L. Patrick: *A model of asynchronous iterative algorithms for solving large, sparse, linear systems.* ICASE Report No. 84-34, July 31, 1984, 27 pages. Proc. 1984 International Conference on Parallel Processing, Robert M. Keller, ed., IEEE Computer Society Press, Silver Spring, MD, 1984, pp. 402-409.

Solving large, sparse, linear systems of equations is one of the fundamental problems in large scale scientific and engineering computation. A model of a general class of asynchronous, iterative solution methods for linear systems is developed. In the model, the system is solved by creating several cooperating tasks that each compute a portion of the solution vector. This model is then analyzed to determine the expected intertask data transfer and task computational complexity as functions of the number of tasks. Based on the analysis, recommendations for task partitioning are made. These recommendations are a function of the sparseness of the linear system, its structure (i.e., randomly sparse or banded), and dimension.

Reed, Daniel A. and Merrell L. Patrick: *Parallel, iterative solution of sparse linear systems: Models and architectures.* ICASE Report No. 84-35, August 1, 1984, 45 pages. Submitted to Parallel Computing.

Solving large, sparse, linear systems of equations is a fundamental problem in large scale scientific and engineering computation. A model of a general class of asynchronous, iterative solution methods for linear systems is developed. In the model, the system is solved by creating several cooperating tasks that each compute a portion of the solution vector. A data transfer model predicting both the probability that data must be transferred between two tasks and the amount of data to be transferred is presented. This model is used to derive an execution time model for predicting parallel execution time and an optimal number of tasks given the dimension and sparsity of the coefficient matrix and the costs of computation, synchronization, and communication.

The suitability of different parallel architectures for solving randomly sparse linear systems is discussed. Based on the complexity of task scheduling, one parallel architecture, based on a broadcast bus, is presented and analyzed.

Tadmor, Eitan: *The well-posedness of the Kuramoto-Sivashinsky equation.* ICASE Report No. 84-36, August 7, 1984, 22 pages. Submitted to SIAM J. Appl. Math.

The Kuramoto-Sivashinsky equation arises in a variety of applications, among which are modeling reaction-diffusion systems, flame-propagation and viscous flow problems. It is considered here, as a prototype to the larger class of generalized Burgers equations: those consist of quadratic nonlinearity and arbitrary linear parabolic part. We show that such equations are well-posed, thus admitting a unique smooth solution, continuously dependent on its initial data. As an attractive alternative to standard energy methods, existence and stability are derived in this case, by "patching" in the large short time solutions without "loss of derivatives."

Lustman, Liviu: *The time evolution of spectral discretizations of hyperbolic systems.* ICASE Report No. 84-37, August 7, 1984, 12 pages. Submitted to SIAM J. Numer. Anal.

A Chebyshev collocation spectral method, applied to hyperbolic systems is considered, particularly for those initial boundary value problems which possess only solutions tending to zero at large times. It is shown that the numerical solutions of the system will also vanish at infinity, if and only if, the numerical solution of a scalar equation of the same type does. This result is then generalized for other spectral approximations.

Bayliss, A., C. I. Goldstein, and E. Turkel: *On accuracy conditions for the numerical computation of waves.* ICASE Report No. 84-38, August 9, 1984, 16 pages. Submitted to J. Comput. Phys.

The Helmholtz equation

$$(\Delta + K^2n^2)u = f$$

with a variable index of refraction n, and a suitable radiation condition at infinity serves as a model for a wide variety of wave propagation problems. Such problems can be solved numerically by first truncating the given unbounded domain and imposing a suitable outgoing radiation condition on an artificial boundary and then solving the resulting problem on the bounded domain by direct discretization (for example, using a finite element method). In practical applications, the mesh size h and the wave number K, are not independent but are constrained by the accuracy of the desired computation. It will be shown that the number of points per wavelength, measured by $(Kh)^{-1}$, is not sufficient to determine the accuracy of a given discretization. For example, the quantity K^3h^2 is shown to determine the accuracy in the L^2 norm for a second-order discretization method applied to several propagation models.

Hariharan, S. I.: *Numerical solutions of acoustic wave propagation problems using Euler computations.* ICASE Report No. 84-39, August 10, 1984, 29 pages. Proc. of the AIAA Ninth Aeroacoustics Conference, Williamsburg, VA, August 15-17, 1984.

This paper reports solution procedures for problems arising from the study of engine inlet wave propagation. The first problem is the study of sound waves radiated from cylindrical inlets. The second one is a quasi-one-dimensional problem to study the effect of nonlinearities and the third one is the study of nonlinearities in two dimensions. In all three problems Euler computations are done with a fourth-order explicit scheme. For the first problem results are shown in agreement with experimental data and for the second problem comparisons are made with an existing asymptotic theory. The third problem is part of an ongoing work and preliminary results are presented for this case.

Tadmor, Eitan: *The exponential accuracy of Fourier and Tchebyshev differencing methods.* ICASE Report No. 84-40, August 20, 1984, 24 pages. Submitted to SIAM J. Numer. Anal.

It is shown that when differencing analytic functions using the pseudospectral Fourier or Tchebyshev methods, the error committed decays to zero at an exponential rate.

Gannon, Dennis and John Van Rosendale: *On the impact of communication complexity in the design of parallel numerical algorithms.* ICASE Report No. 84-41, August 22, 1984, 37 pages. To appear in IEEE Trans. on Computers, December 1984.

This paper describes two models of the cost of data movement in parallel numerical algorithms. One model is a generalization of an approach due to Hockney, and is suitable for shared memory multiprocessors where each processor has vector capabilities. The other model is applicable to highly parallel nonshared memory MIMD systems. In the second model, algorithm performance is characterized in terms of the communication network design. Techniques used in VLSI complexity theory are also brought in, and algorithm independent upper bounds on system performance are derived for several problems that are important to scientific computation.

Ito, Kazufumi: *Legendre-Tau approximation for functional differential equations, Part III: Eigenvalue approximations and uniform stability.* ICASE Report No 84-42, August 24, 1984, 34 pages. Proc. Second International Conference on Control Theory for Distributed Parameter Systems and Applications, Vorau, Austria, 1984.

The stability and convergence properties of the Legendre-tau approximation for hereditary differential systems are analyzed. We derive a characteristic equation for the eigenvalues of the resulting approximate system. As a result of this derivation we are able to establish that the uniform exponential stability of the solution semigroup is preserved under approximation. It is the key to obtaining the convergence of approximate solutions of the algebraic Riccati equation in trace norm.

Berger, M. J.: *On conservation at grid interfaces.* ICASE Report No. 84-43, September 7, 1984, 25 pages. Submitted to J. Comput. Phys.

This paper considers the solution of hyperbolic systems of conservation laws on discontinuous grids. In particular, we consider what happens to conservation at grid interfaces. A procedure is presented to derive conservative difference approximations at the grid interfaces for two-dimensional grids which overlap in an arbitrary configuration. The same procedures are applied to compute interface formulas for grids which are refined in space and/or time, and for continuous grids where a switch in the scheme causes the discontinuity.

Osher, S. and S. Chakravarthy: *Very high order accurate TVD schemes.* ICASE Report No. 84-44, September 10, 1984, 61 pages. Submitted to SIAM J. Numer. Anal.

A systematic procedure for constructing semi-discrete families of 2m-1 order accurate, 2m order dissipative, variation diminishing, 2m+1 point bandwidth, conservation form approximations to scalar conservation laws is presented. Here m is any integer between 2 and 8. Simple first-order forward time discretization, used together with any of these approximations to the space derivatives, also results in a fully discrete, variation diminishing algorithm. These schemes all use simple flux limiters, without which each of these fully discrete algorithms is even linearly unstable. Extensions to systems, using a nonlinear field-by-field decomposition are presented, and shown to have many of the same properties as in the scalar case. For linear systems, these nonlinear approximations are variation diminishing, and hence convergent. A new and general criterion for approximations to be variation diminishing is also given. Finally, numerical experiments using some of these algorithms are presented.

Elcrat, A. R. and L. N. Trefethen: *Classical free-streamline flow over a polygonal obstacle.* ICASE Report No. 84-45, September 10, 1984, 61 pages. Submitted to J. Comput. Appl. Math.

In classical Kirchhoff flow, an ideal incompressible fluid flows past an obstacle and around a motionless wake bounded by free streamlines. Since 1869 it has been known that in principle, the two-dimensional Kirchhoff flow over a polygonal obstacle can be determined by constructing a conformal map onto a polygon in the log-hodograph plane. In practice, however, this idea has rarely been put to use except for very simple obstacles, because the conformal mapping problem has been too difficult. This paper presents a practical method for computing flows over arbitrary polygonal obstacles to high accuracy in a few seconds of computer time. We achieve this high speed and flexibility by working with a modified Schwarz-Christoffel integral that maps onto the flow region directly rather than onto the log-hodograph polygon. This integral and its associated parameter problem are treated numerically by methods developed earlier by Trefethen for standard Schwarz-Christoffel maps.

Krause, Egon: *Review of some vortex relations.* ICASE Report No. 84-46, September 12, 1984, 8 pages. Submitted to Computer and Fluids.

The evaluation of the circulation from numerical solutions of the momentum and energy equations is discussed for incompressible and compressible flows. It is shown how artificial damping directly influences the time rate of change of the circulation.

Leuze, Michael: *Parallel triangularization of substructured finite element problems.* ICASE Report No. 84-47, September 13, 1984, 18 pages. Submitted to Linear Algebra Appl.

Much of the computational effort of the finite element process involves the solution of a system of linear equations. The coefficient matrix of this system, known as the global stiffness matrix, is symmetric, positive definite, and generally sparse. An important technique for reducing the time required to solve this system is substructuring or matrix partitioning. Substructuring is based on the idea of dividing a structure into pieces, each of which can then be analyzed relatively independently. As a result of this division, each point in the finite element discretization is either interior to a substructure or on a boundary between substructures. Contributions to the global stiffness matrix from connections between boundary points form the K_{bb} matrix. This paper focuses on the triangularization of a general K_{bb} matrix on a parallel machine.

Tal-Ezer, Hillel: *A pseudospectral Legendre method for hyperbolic equations with an improved stability condition.* ICASE Report No. 84-48, September 14, 1984, 46 pages. Submitted to J. Comput. Phys.

A new pseudospectral method is introduced for solving hyperbolic partial differential equations. This method uses different grid points than previously used pseudospectral methods: in fact the grid points are related to the zeroes of the Legendre polynomials. The main advantage of this method is that the allowable time step is proportional to the inverse of the number of grid points $1/N$ rather than to $1/N^2$ (as in the case of other pseudospectral methods applied to mixed initial boundary value problems). A highly accurate time discretization suitable for these spectral methods is discussed.

Bayliss, A., C. I. Goldstein, and E. Turkel: *The numerical solution of the Helmholtz equation for wave propagation problems in underwater acoustics.* ICASE Report No. 84-49, September 17, 1984, 29 pages. Submitted to J. Comput. Math. Appl.

The Helmholtz Equation

$$(-\Delta - K^2n^2)u = 0$$

with a variable index of refraction, n, and a suitable radiation condition at infinity serves as a model for a wide variety of wave propagation problems. A numerical algorithm has been developed and a computer code implemented that can effectively solve this equation in the intermediate frequency range. The equation is discretized using the finite element method, thus allowing for the modeling of complicated geometries

(including interfaces) and complicated boundary conditions. A global radiation boundary condition is imposed at the far field boundary that is exact for an arbitrary number of propagating modes.

The resulting large, non-selfadjoint system of linear equations with indefinite symmetric part is solved using the preconditioned conjugate gradient method applied to the normal equations. A new preconditioner is developed based on the multigrid method. This preconditioner is vectorizable and is extremely effective over a wide range of frequencies provided the number of grid levels is reduced for large frequencies. A heuristic argument is given that indicates the superior convergence properties of this preconditioner. The relevant limit to analyze convergence is for K increasing and a fixed prescribed accuracy level. The efficiency and robustness of the numerical algorithm is confirmed for large acoustic models, including interfaces with strong velocity contrasts.

Burns, John A., Ito, Kazufumi, Powers, Robert K.: *Chandrasekhar equations and computational algorithms for distributed parameter systems.* ICASE Report No. 84-50, September 20, 1984, 23 pages. Proc. of 23rd Conference on Decision and Control, December 12-14, 1984, Las Vegas, NV.

In this paper we consider the Chandrasekhar equations arising in optimal control problems for linear distributed parameter systems. The equations are derived via approximation theory. This approach is used to obtain existence, uniqueness, and strong differentiability of the solutions and provides the basis for a convergent computation scheme for approximating feedback gain operators. A numerical example is presented to illustrate these ideas.

Rosen, I. G.: *Approximation methods for inverse problems involving the vibration of beams with tip bodies.* ICASE Report No. 84-51, September 26, 1984, 10 pages. Proc. 23rd IEEE Conference on Decisions and Control, Las Vegas, NV, December 12-14, 1984.

We outline two cubic spline based approximation schemes for the estimation of structural parameters associated with the transverse vibration of flexible beams with tip appendages. The identification problem is formulated as a least squares fit to data subject to the system dynamics which are given by a hybrid system of coupled ordinary and partial differential equations. The first approximation scheme is based upon an abstract semigroup formulation of the state equation while a weak/variational form is the basis for the second. Cubic spline based subspaces together with a Rayleigh-Ritz-Galerkin approach was used to construct sequences of easily solved finite dimensional approximating identification problems. Convergence results are briefly discussed and a

numerical example demonstrating the feasibility of the schemes and exhibiting their relative performance for purposes of comparison is provided.

OTHER ACTIVITIES

The summer program for 1984 included the following visitors:

NAME/AFFILIATION	DATE OF VISIT	AREA OF INTEREST
Abarbanel, Saul Tel-Aviv University	6/18 - 9/10	Computational Fluid Dynamics
Banks, H. Thomas Brown University	6/11 - 6/22	Control Theory
Berger, Marsha J. Courant Institute	7/23 - 8/17	Numerical Methods for PDE
Bokhari, Shahid H. University of Engineering and Technology Pakistan	7/2 - 12/31/85 (18 months)	Distributed Computing
Brewer, Dennis W. University of Arkansas	6/11 - 11/30/85 (18 months)	Control Theory
Choudhury, Shenaz Carnegie-Mellon University	5/14 - 6/8	Numerical Methods for PDE
Crowley, James M. U. S. Air Force Academy	6/11 - 6/22	Control Theory
Flandoli, Franco Instituto di Analyse Math Italy	6/11 - 6/22	Riccati Equations
Fleishman, Bernard A. Rensselaer Polytechnic Institute	5/16 - 6/1	Applied Mathematics
Gibson, J. Steven University of California, LA	6/18 - 6/22	Control Theory
Gokhale, Maya University of Delaware	6/25 - 7/6	Programming Languages
Gottlieb, David Tel-Aviv University	6/12 - 9/28	Numerical Methods for PDE
Gunzburger, Max D. Carnegie-Mellon University	5/7 - 6/1	Numerical Methods for PDE

NAME/AFFILIATION	DATE OF VISIT	AREA OF INTEREST
Hariharan, S. I. University of Tennessee Space Institute	6/11 - 8/17	Acoustics
Harten, Amiram Tel-Aviv University	8/3 - 8/10	Partial Differential Equations
Hirsch, Charles Vrije Universiteit Brussel, Belgium	7/31 - 8/30	Computational Fluid Dynamics
Hockney, Roger W. University of Reading, England	7/9 - 8/31	Algorithms for Parallel Array Computers
Ipsen, Ilse Research Center for Scientific Computation, Yale University	5/30 - 7/6	Algorithms for Parallel Systems
Israeli, Moshe Technion, Israel Institute of Technology	7/84 & 9/84	Computational Fluid Dynamics
Krause, Egon Aerodynamisches Institute der RWTH, West Germany	8/31 - 9/14	Computational Fluid Dynamics
Kumar, Swarn P. Colorado State University	7/2 - 7/13	Parallel Methods for Computational Fluid Dynamics
Lamm, Patricia D. Southern Methodist University	6/11 - 6/15	Control Theory
Leuze, Michael Vanderbilt University	7/23 - 7/27	Parallel Methods for Linear Algebra
LeVeque, Randall J. University of California, LA	6/25 - 8/24	Numerical Methods for PDE
McCormick, Stephen F. University of Colorado, Denver	7/23 - 8/3	Multigrid Methods
MacCormack, Robert W. University of Washington	7/23 - 8/10	Computational Fluid Dynamics

NAME/AFFILIATION	DATE OF VISIT	AREA OF INTEREST
Maday, Yvon Universite Pierre et Marie Currie, France	6/25 - 7/13	Partial Differential Equations
Majda, George J. Brown University	8/13 - 8/31	Numerical Methods for PDE
Murphy, Katherine A. Brown University	6/11 - 6/22	Control Theory
Napolitano, Michele Istituto di Macchine Italy	8/13 - 8/17	Computational Fluid Dynamics
Nicolaides, R. A. Carnegie-Mellon University	5/14 - 6/1	Finite Element Methods
Osher, Stanley J. University of California, LA	7/23 - 8/3	Numerical Methods for PDE
Patrick, Merrell L. Duke University	4/30 - 8/3	Algorithms for Parallel Array Computers
Pratt, Terrence W. University of Virginia	5/28 - 6/22 7/9 - 7/20	Programming Languages
Reed, Daniel A. University of North Carolina	5/7 - 6/8 7/10 - 7/13	Performance of Parallel and Distributed Systems
Rodriguez, Guillermo California Institute of Technology	6/11 - 6/15	Control Theory
Roe, Phillip L. Royal Aircraft Establishment England	7/2 - 8/3	Computational Fluid Dynamics
Rosen, I. Gary Charles Stark Draper Laboratory, Inc.	6/18 - 6/22	Numerical Methods for Problems in Control Systems
Ruge, John Colorado State University	7/23 - 8/3	Multigrid Methods
Salamon, Dietmar University of Wisconsin	6/11 - 6/15	Control Theory
Saltz, Joel Duke University	5/21 - 8/24	Parallel Algorithms

NAME/AFFILIATION	DATE OF VISIT	AREA OF INTEREST
Seinfeld, John H. California Institute of Technology	6/11 - 6/15	Control Theory
Strikwerda, John C. University of Wisconsin	8/13 - 8/17	Numerical Methods for PDE
Tadmor, Eitan Tel-Aviv University	6/11 - 9/25	Numerical Methods for PDE
Tal-Ezer, Hillel Tel-Aviv University	7/5 - 9/14	Numerical Methods for PDE
Tran, Hien T. Rensselaer Polytechnic Institute	6/11 - 6/22	Control Theory
Trefethen, Lloyd N. Courant Institute	7/23 - 8/3	Numerical Methods for PDE
Turkel, Eli Tel-Aviv University	7/2 - 9/20	Computational Fluid Dynamics
Turner, James C. Carnegie-Mellon University	5/14 - 6/8	Numerical Methods for PDE
van Leer, Bram Delft University of Technology, Netherlands	7/16 - 9/3	Computational Fluid Dynamics
White, Luther W. University of Oklahoma	6/11 - 6/15	Control Theory
Woodward, Paul R. Lawrence Livermore National Laboratory	5/7 - 5/17	Numerical Methods for PDE

On October 10-12, ICASE will host a small workshop to explore the various approaches used to study turbulence. Approximately 15 researchers have been invited to speak and their papers will be collected into a volume published by Springer.

ICASE STAFF

April 1, 1984 through September 30, 1984

I. ADMINISTRATIVE

Milton E. Rose, Director
Ph.D., Mathematics, New York University, 1953
Numerical Methods

Robert G. Voigt, Associate Director
Ph.D., Mathematics, University of Maryland, 1969
Numerical and Computational Techniques

Linda T. Johnson, Administrative Assistant

Georgia V. Ballance, Technical Publications/Summer Housing Secretary

Etta M. Blair, Office Assistant (Beginning April 1984)

Barbara A. Kraft, Senior Technical Publications Secretary (Beginning May 1984)

Barbara A. Rohrbach, Office Assistant
Personnel/Bookkeeping Secretary (Beginning June 1984)

Susan B. Ruth, Personnel/Bookeeping Secretary (Through June 1984)

Emily N. Todd, Visitor Coordinator/Correspondence Secretary

II. SCIENCE COUNCIL for APPLIED MATHEMATICS and COMPUTER SCIENCE

Bruce Arden, Chairman and Arthur Doty Professor, Department of Electrical Engineering and Computer Science, Princeton University.

Michael J. Flynn, Professor, Department of Electrical Engineering, Computer Systems Laboratory, Stanford University.

Bernard Galler, Professor, Department of Computer and Communication Sciences and Associate Director of the Computer Center, University of Michigan.

C. William Gear, Professor, Department of Computer Science, University of Illinois at Urbana.

Anthony C. Hearn, Department Head, Department of Information Sciences, Rand Corporation.

Seymour V. Parter, Professor, Department of Mathematics, University of Wisconsin.

Werner C. Rheinboldt, Andrew W. Mellon Professor, Department of Mathematics and Statistics, University of Pittsburgh.

III. ASSOCIATE MEMBERS

Saul S. Abarbanel, Professor, Department of Applied Mathematics, Tel-Aviv University, Israel.

Herbert E. Keller, Professor, Department of Applied Mathematics, California Institute of Technology.

Peter D. Lax, Professor, Courant Institute of Mathematical Sciences, New York University.

William R. Sears, Professor, Department of Aerospace and Mechanical Engineering, University of Arizona.

IV. SENIOR STAFF SCIENTISTS

David Gottlieb - Ph.D., Numerical Analysis, Tel-Aviv University, Israel, 1972. Professor and Chairman, Department of Applied Mathematics, Tel-Aviv University. Numerical Methods for Partial Differential Equations. (July 1974 to January 1987)

M. Yousuff Hussaini - Ph.D., Mechanical Engineering, University of California, 1970. Computational Fluid Dynamics. (April 1978 to January 1987)

V. SCIENTIFIC STAFF

Loyce M. Adams - Ph.D., Applied Mathematics, University of Virginia, 1983. Parallel Numerical Algorithms. (October 1982 to August 1984)

Stephen F. Davis - Ph.D., Applied Mathematics, Rensselaer Polytechnic Institute, 1980. Numerical Methods for Partial Differential Equations. (February 1982 to September 1985)

Murshed Hossain - Ph.D., Physics, College of William and Mary, 1983. Fluid Turbulence. (December 1983 to December 1985)

Kazufumi Ito - Ph.D., Systems Science and Mathematics, Washington University, 1981. Control Theory. (August 1981 to August 1984)

David A. Kopriva - Ph.D., Applied Mathematics, University of Arizona, 1982. Computational Fluid Dynamics. (July 1982 to August 1984)

Liviu Lustman - Ph.D., Applied Mathematics, Tel-Aviv University, Israel, 1979. Numerical Methods for Partial Differential Equations, Spectral Methods, Computational Fluid Dynamics. (August 1984 to September 1986)

Piyush Mehrotra - Ph.D., Computer Science, University of Virginia, 1982. Language Concepts for Parallel and Distributed Systems. (July 1983 to August 1984)

Vijay Naik - MS., Mechanical Engineering, University of Florida, 1982. AM., Computer Science, Duke University, 1984. Parallel Processing, Computational Fluid Dynamics. (August 1984 to August 1986)

Timothy Phillips - Ph.D., Numerical Analysis, Oxford University, United Kingdom, 1983. Numerical Solution of Elliptic Partial Differential Equations. (October 1982 to September 1984)

Robert K. Powers - Ph.D., Applied Mathematics, Virginia Polytechnic Institute and State University, 1984. Numerical Techniques for Optimal Control. (March 1984 to September 1985)

Nancy E. Shoemaker - Ph.D., Mathematics, Rensselaer Polytechnic Institute, 1977. Computer Systems, particularly UNIX. (February 1982 to October 1987)

Sivaguru S. Sritharan - Ph.D., Applied Mathematics, University of Arizona, 1982. Computational Fluid Dynamics. (July 1982 to June 1984)

John Van Rosendale - Ph.D., Computer Science, University of Illinois, 1980. Numerical Solution of Partial Differential Equations. (June 1981 to June 1985)

VI. VISITING SCIENTISTS

Saul S. Abarbanel - Ph.D., Theoretical Aerodynamics, Massachusetts Institute of Technology, 1959. Professor, Department of Applied Mathematics, Tel-Aviv University, Israel. Numerical Analysis of Partial Differential Equations. (January to December 1984)

Dennis W. Brewer - Ph.D., Applied Mathematics, University of Wisconsin-Madison, 1975. Associate Professor, Mathematical Sciences, University of Arkansas. Control Theory. (June 1984 to August 1985)

Shahid H. Bokhari - Ph.D., Electrical and Computer Engineering, University of Massachusetts, Amherst, 1978. Associate Professor, Department of Electrical Engineering, University of Engineering and Technology, Lahore, Pakistan. Parallel Processing, Distributed Computing and Computer Architecture. (July 1984 to June 1986)

Franco Flandoli - Ph.D., Mathematics, University of Pisa, 1983. Researcher, Department of Mathematics, University of Torino, Italy. Control Theory, Riccati Equations. (June 1984)

Bernard A. Fleishman - Ph.D., Mathematics, New York University, 1952. Professor, Department of Applied Mathematics, Rensslaer Polytechnic Institute. Applied Mathematics. (May - June 1984)

Maya Gokhale - Ph.D., Computer Science, University of Pennsylvania, 1983. Assistant Professor, Department of Computer Science, University of Delaware. Nonprocedural Languages and Parallel Processing. (June - July 1984)

Max D. Gunzburger - Ph.D., Mathematics, New York University, 1969. Professor, Department of Mathematics, Carnegie-Mellon University. Fluid Dynamics and Numerical Methods for Partial Differential Equations. (May - June 1984)

Philip Hall - Ph.D., Mathematics, Imperial College, London, England, 1973. Lecturer, Department of Mathematics, Imperial College, London, England. Computational Fluid Dynamics. (March 1983 to September 1984)

Subramaniya I. Hariharan - Ph.D., Mathematics, Carnegie-Mellon University, 1980. Assistant Professor, University of Tennessee Space Institute. Numerical Methods for Partial Differential Equations. (June - August 1984)

Ami Harten - Ph.D., Mathematics, Courant Institute, 1974. Associate Professor, Department of Mathematics, Tel-Aviv University, Israel. Numerical Solution for Partial Differential Equations. (August 1984)

Charles Hirsch - Ph.D., Aeronautics, University of Brussel, 1961. Professor and Department Head, Department of Fluid Mechanics, Vrije Universiteit Brussel. Computational Fluid Dynamics. (August 1984)

Roger W. Hockney - Ph.D., Numerical Analysis and Plasma Physics, Stanford University, 1966. Professor, Department of Computer Science, Reading University, England. Algorithms for Parallel Array Computers. (July - August 1984)

Ilse Ipsen - Ph.D., Computer Science, Pennsylvania State University, 1983. Research Associate, Department of Computer Science, Yale University. Parallel Computations in Numerical Linear Algebra. (June - July 1984)

Moshe Israeli - Ph.D., Applied Mathematics, Massachusetts Institute of Technology, 1971. Associate Professor, Department of Computer Science, Technion - Israel Institute of Technology. Computational Fluid Dynamics. (July and September 1984)

Egon Krause - Ph.D., Aeronautics, New York University, 1966. Professor, Department of Fluid Dynamics, Aerodynamisches Institut der RWTH, Germany. Computational Fluid Dynamics. (September 1984)

Swarn P. Kumar - Ph.D., Computer Science, Washington State University, 1982. Assistant Professor, Department of Computer Science, Colorado State University. Parallel Methods for Computational Fluid Dynamics. (July 1984)

Michael R. Leuze - Ph.D., Computer Science, Duke University, 1981. Assistant Professor, Vanderbilt University. Parallel Methods for Linear Algebra. (August 1984)

Randall J. LeVeque - Ph.D., Computer Science, Stanford University, 1982. Assistant Professor, Department of Applied Mathematics, University of California at Los Angeles. Numerical Methods for Partial Differential Equations. (June - August 1984)

Stephen F. McCormick - Ph.D., Mathematics, University of Southern California, 1971. Professor, Department of Applied Mathematics, University of Colorado at Denver. Multigrid Methods. (July - August 1984)

Yvon Maday - Ph.D., Numerical Analysis, University of Paris, VI, 1981. Assistant Professor, University of Paris, France. Spectral Methods for Partial Differential Equations. (June - July 1984)

George J. Majda - Ph.D., Mathematics/Numerical Analysis, New York University, 1980. Assistant Professor, Division of Applied Mathematics, Brown University. Numerical Methods for Partial Differential Equations. (August 1984)

Michele Napolitano - Ph.D., Computational Fluid Dynamics, University of Cincinnati, 1978. Professor, Istituto di Maccaine, Universita di Bari, Italy. Numerical Methods for Partial Differential Equations. (August 1984)

Roy A. Nicolaides - Ph.D., Mathematics, University of London, 1972. Professor, Department of Mathematics, Carnegie-Mellon University. Numerical Solution of Partial Differential Equations. (May - June 1984)

Robert E. Noonan - Ph.D., Computer Science, Purdue University, 1971. Professor, Department of Computer Science, College of William and Mary. Computer Systems, Multi-Processors. (September 1984 - May 1985)

Stanley Osher - Ph.D., Functional Analysis, New York University, 1966. Professor, Department of Mathematics, University of California at Los Angeles. Methods for the Numerical Analysis of Partial Differential Equations. (July - August 1984)

Merrell L. Patrick - Ph.D., Mathematics, Carnegie-Mellon University, 1964. Professor, Department of Computer Science, Duke University. Parallel Numerical Algorithms and Architectures. (April - August 1984)

Terrence W. Pratt - Ph.D., Mathematics/Computer Science, University of Texas at Austin, 1965. Professor, Department of Computer Science, University of Virginia. Programming Languages. (May - June 1984)

Daniel A. Reed - Ph.D., Computer Science, Purdue University, 1983. Assistant Professor, Department of Computer Science, University of North Carolina. Parallel Processing. (May - June 1984)

Guillermo Rodriguez - Ph.D., Control Theory, University of California at Los Angeles, 1974. Technical Group Supervisor, Automated Systems, Jet Propulsion Laboratory, California Institute of Technology. Numerical Methods for Parameter Estimaton. (June 1984)

Philip L. Roe - Ph.D., Cambridge University, 1962. Professor, Department of Computational Fluid Dynamics, Cranfield Institute of Technology, England. Computational Fluid Dynamics. (July - August 1984)

John Ruge - Ph.D., Mathematics, Colorado State University, 1981. Research Scientist, Research Institute of Colorado. Multigrid Methods. (July - August 1984)

Dietmar Salamon - Ph.D., Mathematics, University of Bremen, 1982. Research Associate, University of Wisconsin-Madison. Control Theory. (June 1984)

John H. Seinfeld - Ph.D., Chemical Engineering, Princeton University, 1967. Professor, Department of Chemical Engineering, California Institute of Technology. Control Theory. (June 1984)

Eitan Tadmor - Ph.D., Numerical Analysis, Tel-Aviv University, 1979. Senior Lecturer, Department of Applied Mathematics, Tel-Aviv University, Israel. Numerical Methods for Partial Differencial Equations. (June - September 1984)

Lloyd N. Trefethen - Ph.D., Computer Science, Stanford University, 1982. Assistant Professor, Department of Applied Mathematics, Massachusettes Institute of Technology. Numerical Methods for Partial Differential Equations. (July - August 1984)

Eli Turkel - Ph.D., Applied Mathematics, New York University, 1970. Associate Professor, Department of Mathematics, Tel-Aviv University, Israel. Computational Fluid Dynamics. (January - December 1984)

Bram van Leer - Ph.D., Theoretical Astrophysic, Leiden State University, 1970. Research Leader, Department of Applied Mathematics and Computer Science, Delft University of Technology, Netherlands. Computational Fluid Dynamics. (July - September 1984)

Luther W. White - Ph.D., Mathematics, University of Illinois, 1977. Associate Professor, Department of Applied Mathematics, University of Oklahoma. Control Theory. (June 1984)

Paul R. Woodward - Ph.D., Physics, University of California, 1973. Computational Physicist, Department of Physics, Lawrence Livermore National Laboratory. Numerical Methods for Partial Differential Equations. (May 1984)

VII. CONSULTANTS

Ivo Babuska - Ph.D., Technical Sciences, Tech University, Prague, Czechoslovakia, 1952; Mathematics, Academy of Sciences, Prague, 1956; D.Sc., Mathematics, Academy of Science, Prague, 1960. Professor, Institute for Physical Science and Technology, University of Maryland. Numerical Methods for Partial Differential Equations.

H. Thomas Banks - Ph.D., Applied Mathematics, Purdue University, 1967. Professor, Division of Applied Mathematics, Brown University. Control Theory and Estimation of Parameters, Mathematical Biology, Functional and Partial Differential Equations.

Alvin Bayliss - Ph.D., Mathematics, New York University, 1975. Senior Staff Scientist, Exxon Corporate Research. Numerical Methods for Partial Differential Equations.

Marsha J. Berger - Ph.D., Numerical Analysis, Stanford University, 1982. Research Associate, Courant Institute of Mathematical Sciences. Numerical Methods for Partial Differential Equations.

James M. Crowley - Ph.D., Applied Mathematics, Brown University, 1982. Assistant Professor, Department of Mathematics, (Captain), U.S. Air Force Academy. Identification for Distributed Parameter Systems.

Peter R. Eiseman - Ph.D., Mathematics, University of Illinois, 1970. Senior Research Scientist and Adjunct Professor, Department of Applied Physics and of Nuclear Engineering, Columbia University. Computational Fluid Dynamics.

Stefan Feyock - Ph.D., Computer Science, University of Wisconsin, 1971. Associate Professor, Department of Mathematics and Computer Science, College of William and Mary. Artificial Intelligence.

George J. Fix - Ph.D., Mathematics, Harvard University, 1968. Professor and Chairman, Department of Mathematics, Carnegie-Mellon University. Numerical Methods for Partial Differential Equations.

Joseph E. Flaherty - Ph.D., Applied Mechanics, Polytechnic Institute of Brooklyn, 1969. Professor and Chairman, Departments of Mathematical Sciences and Computer Science, Rensselaer Polytechnic Institute. Singular Perturbations, Numerical Analysis and Applied Mathematics.

Dennis B. Gannon - Ph.D., Computer Science, University of Illinois, 1980. Assistant Professor, Department of Computer Science, Purdue University. Numerical Methods and Software and Architecture Design.

James F. Geer - Ph.D., Applied Mathematics, New York University, 1967. Professor, Systems Science and Mathematical Sciences, Watson School of Engineering, Applied Science and Technology, SUNY-Binghamton. Pertubation Methods and Asymptotic Expansions of Solutions to Partial Differential Equations.

J. Steven Gibson - Ph.D., Engineering Mechanics, University of Texas at Austin, 1975. Associate Professor, Department of Mechanical, Aerospace and Nuclear Engineering, University of California at Los Angeles. Control of Distributed Systems.

Maya Gokhale - Ph.D., Computer Science, University of Pennsylvania, 1983. Assistant Professor, Department of Computer Science, University of Delaware. Nonprocedural Languages and Parallel Processing.

Chester E. Grosch - Ph.D., Physics - Fluid Dynamics, Stevens Institute of Technology, 1967. Professor, Department of Computer Science and Slover Professor, Department of Oceanography, Old Dominion University. Hydrodynamic Stability, Computational Fluid Dynamics, Unsteady Boundary Layers and Algorithms for Array Processors.

Max D. Gunzburger - Ph.D., Mathematics, New York University, 1969. Professor, Department of Mathematics, Carnegie-Mellon University. Fluid Dynamics and Numerical Methods for Partial Differential Equations.

Subramaniya I. Hariharan - Ph.D., Mathematics, Carnegie-Mellon University, 1980. Assistant Professor, University of Tennessee Space Institute. Numerical Methods for Partial Differential Equations.

Ami Harten - Ph.D., Mathematics, Courant Institute, 1974. Associate Professor, Department of Mathematics, Tel-Aviv University, Israel. Numerical Solution for Partial Differential Equations.

Harry F. Jordan - Ph.D., Physics, University of Illinois, 1977. Professor Department of Electrical and Computer Engineering, University of Colorado at Boulder. Parallel Computation.

Ashwani K. Kapila - Ph.D., Theoretical and Applied Mechanics, Cornell University, 1975. Associate Professor, Department of Mathematical Sciences, Rensselaer Polytechnic Institute. Ordinary and Partial Differential Equations, Asymptotic Methods.

Heinz-Otto Kreiss - Ph.D., Mathematics, Royal Institute of Technology, Sweden 1960. Professor, Department of Applied Mathematics, California Institute of Technology. Numerical Analysis.

William D. Lakin - Ph.D., Applied Mathematics, University of Chicago, 1968. Eminent Professor, Department of Mathematical Sciences, Old Dominion University. Fluid Mechanics and Elastic Vibrations.

Patricia Daniel Lamm - Ph.D., Applied Mathematics, Brown University, 1981. Assistant Professor, Department of Mathematics, Southern Methodist University. Control and Identification of Partial Differential Equations.

Robert W. MacCormack - M.S., Mathematics, Stanford University. Professor, Department of Aeronautics and Astronautics, University of Washington. Computational Fluid Dynamics and Numerical Analysis.

David C. Montgomery, Ph.D., Physics, Princeton, 1959. Professor, Department of Physics and Astronomy, Dartmouth College. Plasma Physics, Turbulence Theory and Magnetohydrodynamics.

William F. Moss - Ph.D., University of Delaware, 1974. Associate Professor, Department of Mathematical Sciences, Clemson University. Integral Equation Methods, Nonlinear Eigenanalysis.

Katherine A. Murphy - Ph.D., Applied Mathematics, Brown University, 1983. Visiting Assistant Professor, Department of Applied Mathematics, Brown University. Control Theory and Estimation of Parameters.

Roy A. Nicolaides - Ph.D., Mathematics, University of London, 1972. Professor, Department of Mathematics, Carnegie-Mellon University. Numerical Solution of Partial Differential Equations.

Joseph E. Oliger - Ph.D., Numerical Analysis, Uppsala University, Sweden, 1973. Associate Professor, Department of Computer Science, Stanford University. Numerical Methods for Partial Differential Equations.

James E. Ortega - Ph.D., Mathematics, Stanford University, 1962. Professor and Chairman, Department of Applied Mathematics, University of Virginia. Numerical Methods for Partial Differential Equations.

Stanley Osher - Ph.D., Functional Analysis, New York University, 1966. Professor, Department of Mathematics, University of California at Los Angeles. Methods for the Numerical Analysis of Partial Differential Equations.

Anthony Patera - Ph.D., Applied Mathematics, Massachusetts Institute of Technology, 1982. Assistant Professor, Department of Mechanical Engineering, Massachusetts Institute of Technology. Numerical Methods for Fluid Dynamics and Heat Transfer.

Merrell L. Patrick - Ph.D., Mathematics, Carnegie-Mellon University, 1964. Professor, Department of Computer Science, Duke University. Parallel Numerical Algorithms and Architectures.

Terrence W. Pratt - Ph.D., Mathematics/Computer Science, University of Texas at Austin, 1965. Professor, Department of Computer Science, University of Virginia. Programming Languages.

Daniel A. Reed - Ph.D., Computer Science, Purdue University, 1983. Assistant Professor, Department of Computer Science, University of North Carolina. Parallel Processing.

Werner C. Rheinboldt - Ph.D., Applied Mathematics, University of Freiburg, Germany, 1955. Andrew W. Mellon Professor, Department of Mathematics and Statistics, University of Pittsburgh. Numerical Analysis and Computational Solution of Nonlinear Problems.

I. Gary Rosen - Ph.D., Applied Mathematics, Brown University, 1980. Assistant Professor, Department of Applied Mathematics, University of Southern California, Numerical Methods for Problems in Control Systems.

Jacob T. Schwartz - Ph.D., Mathematics, Yale University, 1953. Professor, Department of Computer Science, Courant Institute for Mathematical Sciences. Programming Languages, Parallel Computing and Artificial Intelligence.

John C. Strikwerda - Ph.D., Mathematics, Stanford University, 1976. Associate Professor, Department of Computer Sciences, University of Wisconsin-Madison. Numerical Methods for Partial Differential Equations.

George M. Vahala - Ph.D., Physics, University of Iowa, 1972. Associate Professor, Department of Physics, College of William and Mary. Plasma Physics, Magnetohydrodynamics, Nonlinear Dynamics, Turbulence.

J. Christian Wild - Ph.D., Computer Science, Rutgers University, 1977. Assistant Professor, Department of Computer Science, Old Dominion University. Concurrent Computing Systems.

VIII. STUDENT ASSISTANTS

Mary Ann O. Bynum - Graduate student at the College of William and Mary. (September 1984 to Present)

Annette D. Holloway - Graduate student at University of Virginia. (June - August 1984)

Mala Mehrotra - Graduate student at the College of William and Mary. (July 1983 to August 1984)

Eiichi Oka - Graduate student at Old Dominion University. (February 1984 to present)

James M. Stanley - Graduate student at the College of William and Mary. (January - August 1984)

GRADUATE FELLOWS

Shenaz Choudhury - Student at Carnegie-Mellon University. (May - June 1984)

Joel H. Saltz - Student at Duke University. (May - August 1984)

Hillel Tal-Ezer - Student at Tel-Aviv University, ISRAEL. (July - September 1984)

Hien T. Tran - Student at Rensselaer Polytechnic Institute. (June 1984)

James C. Turner - Student at Carnegie-Mellon University. (May - June 1984)

ICASE SEMINAR PROGRAM

April 1, 1984 through September 30, 1984

April 2 — **Dr. Ali Eghlima**, Rensselaer Polytechnic Institute: *Thick Turbulent Shear Layer Computations*

April 6 — **Dr. Philip Marcus**, Harvard University: *Pseudo Spectral Methods for Flow Between Rotating Cylinders*

April 9 — **Mr. Chung-Teh Wu**, Stanford University: *Numerical Simulation of Homogeneous, Compressed Turbulence*

April 12 — **Professor Maya Gokhale**, University of Delaware: *Automatic Generation of Parallel Programs from Nonprocedural Specifications*

April 16 — **Dr. Michele Macaraeg**, NASA Langley Research Center: *Numerical Model of the Axisymmetric Flow in a Heated, Rotating Spherical Shell*

April 19 — **Dr. Daniel M. Nosenchuck**, Princeton University: *The Use of Special-Purpose Computers in Fluid Mechanics*

April 20 — **Dr. Philip Hall**, ICASE: *The Gortler Vortex Instability Mechanism in Three-Dimensional Boundary Layers*

May 2 — **Professor George Vahala**, The College of William and Mary: *Spectral Properties of Magnetohydrodynamics*

May 9 — **Mr. Alan D. Geller**, *ELXSI: The ELXSI 6400 System*

May 15 — **Dr. Paul Woodward**, Lawrence Livermore National Laboratory: *Simulations of Supersonic Kelvin-Helmholtz Instability with the PPM Hydrocode*

May 16 — **Dr. John W. Murdock**, Aerospace Corporation: *Numerical Simulation of Transitional and Turbulent Boundary Layers*

May 17 **Professor Ashwani Kapila,** Rensselaer Polytechnic Institute: *Some Problems in Flame Theory*

May 21 **Dr. M. Calvin Mosher,** David W. Taylor Naval Ship Research and Development Center: *A Method for Computing Three-Dimensional Vortex Flows*

June 6 **Dr. Stanley A. Berger,** University of California, Berkeley: *Studies in Vortex Breakdown*

June 7 **Dr. Stanley A. Berger,** University of California, Berkeley: *Instability and Break-Up of a Liquid Jet Treated as an Initial-Value Problem*

June 12 **Ms. Jill Dahlburg,** College of William and Mary: *Numerical Simulation of Turbulent Disruptions from the Strauss Equations*

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September 25 **Professor Claudio Canuto**, Istituto di Analisi Numerica del C.N.R., Italy: *Non-Dirichlet Boundary Conditions in Legendre and Chebyshev Methods*

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15. Supplementary Notes

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16. Abstract

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